SCHEDULE

Monday, June 20

- 09.45–10.15: Registration
- 10.15–10.30: Welcome (Helene Barcelo (MSRI), Charalambos Macridakis (FORTH))
- 10.30–11.30: Pre-lecture:
  Review of curvature, commutator identities for covariant derivatives
- 11.30-12.00: Coffee break
- 12.00–13.15: Simon Brendle on **Ricci flow**:
  Definition of Ricci flow, some basic examples, self-similar solutions, evolution of curvature, curvature algebra in three dimensions
- 13.15–13.45: Informal questions (TAs)*
- 13.45–15.30: Lunch break
- 15.30–16.30: Post-lecture / Problem session

Tuesday, June 21

- 09.45–10.45: Pre-lecture:
  Heat equation on $\mathbb{R}^n$ and on manifolds
- 10.45-11.15: Coffee break
- 11.15–12.30: Panagiota Daskalopoulos on **Ancient solutions to parabolic equations**:
  Ancient solutions to the heat equation and to the semi-linear heat equation
- 12.30-13.00: Informal questions (TAs)*
- 13.00-15.00: Lunch break
- 15.00–16.00: Post lecture/Problem session
- 16.00–17.00: Informal discussions between students**

Wednesday, June 22

- 09.45–10.45: Pre-lecture
  Maximum principle for scalar heat equations, interior estimates
- 10.45–11.15: Coffee break
- 11.15–12.30: Simon Brendle on **Ricci flow**:
  Hamilton's maximum principle for systems, preservation of nonnegative Ricci in three dimensions, pinching and convergence in three dimensions, curvature conditions in higher dimensions
- 12.30–13.00: Informal questions (TAs)*
- 13.00–15.00: Lunch break
- 15.00–16.00: Post-lecture and problem session
- 16.00–17.00: Informal discussions between students**

Thursday, June 23

- 09.45–10.45: Pre-lecture
  Log-Sobolev entropy and Nash entropy
- 10.45–11.15: Coffee break
- 11.00–12.30: Simon Brendle on **Ricci flow**:
  Perelman's entropy, and noncollapsing
- 12.30-13.00: Informal questions (TAs)*
- 13.00-15.00: Lunch break
- 15.00–16.00: Post-lecture / Problem session
- 16.00–17.00: Informal discussions between students**

Friday, June 24

- 09.45–10.45: Pre-lecture:
  Extrinsic curvature of hypersurfaces, and evolution equations for extrinsic geometric flows
- 10.45–11.15: Coffee break
- 11.15–12.30: Gerhard Huisken on **Inverse Mean Curvature Flow**:
  Properties of smooth solutions to IMCF
- 12.30-13.00: Informal questions (TAs)*
- 13.00-15.00: Lunch break
- 15.00–16.00: Post-lecture / Problem session
- 16.00–17.00: Informal discussions between students**

Monday, June 27

- 09.45–10.45: Pre-lecture:
  Level set approach to extrinsic geometric flows (and the relation to the parametrized version of the flow); weak mean curvature
- 11.00–12.15: Gerhard Huisken on **Inverse Mean Curvature Flow:**
  Weak solutions for inverse mean curvature flow
- 12.30-13.00: Informal questions (TAs)*
- 13.00-15.00: Lunch break
- 15.00–16.00: Post-lecture / Problem session
- 16.00–17.00: Informal discussions between students**

Tuesday, June 28

- 09.45–10.45: The 2-dimensional Ricci flow
- 10.45–11.15: Coffee break
- 11.15–12.30: Panagiota Daskalopoulos on **Ancient solutions to geometric flows:**
  Ancient solutions to the 2-dim Ricci flow
- 12.30-13.00: Informal questions (TA's)*
- 13.00-15.00: Lunch break
- 15.00–16.00: Post-lecture / Problem session
- 16.00–17.00: Informal discussions between students**

Wednesday, June 29

- 09.45–10.45: Pre-lecture:
  Asymptotically flat 3-manifolds, their role in General Relativity, the ADM-mass and the dominant energy condition
- 10.45–11.15: Coffee break
- 11.00–12.30: Gerhard Huisken on **Inverse Mean Curvature Flow**
  Applications to General Relativity
- 12.30-13.00: Informal questions (TAs)*
- 13.00-15.00: Lunch break
- 15.00–16.00: Post-lecture / Problem session
- 16.00–17.00: Informal discussions between students**

Thursday, June 30

- 09.45–10.45: Pre-lecture
  Curve shortening flow on the plane
- 10.45–11.15: Coffee break

- 11.15–12.30: Theodora Bourni on **Ancient solutions to geometric flows:** Ancient solutions to the curve shortening flow.
- 12.30-13.00: Informal questions (TAs)*
- 13.00-15.00: Lunch break
- Afternoon free for discussions
- 16.00–17.00: Informal discussions between students**

*: The Informal questions will take place in the "Discussion classroom".
**: The "Discussion classroom" will be available to the students for these informal discussions. In addition, students are encouraged to use the outdoor space available.

Detailed outline

Lecture series by Gerhard Huisken on **Inverse Mean Curvature Flow**

(1) Properties of smooth solutions to Inverse Mean Curvature Flow (IMCF):
  (a) Evolution for starshaped surfaces (convergence result by C. Gerhardt [2], Harnack inequality by G. Huisken and T. Ilmanen [5])
  (b) Existence theory under lower bound on mean curvature ($H > \delta > 0$)
  (c) Interior upper bound on mean curvature H
  (d) Bound on the norm of second fundamental $|A|$ by M. Heidusch [3] and applications to non-compact surfaces by P. Daskalopoulos and G. Huisken [1]

  Pre-lecture:
  (a) Basic notions from extrinsic geometry (second fundamental form, Gauss equations, Codazzi equations, Simons identity)
  (b) Evolution equations for curvature for any deformation of a hypersurface; see for example the notes by G. Huisken and A. Polden [6]

(2) Weak solutions
  (a) Level-set formulation of IMCF
  (b) Variational concept of weak solutions for IMCF
  (c) Construction of weak solutions using the interior upper bound on the mean curvature
  (d) Properties of weak solutions and anisotropic extensions of IMCF; compare with work by K. Moore [7]; and G. Huisken and M. Wolff

  Pre-lecture:
  (a) Equivalence of level-set formulation and parametric version of flow for strictly positive speeds
  (b) Notion of weak mean curvature
  (c) Allard regularity theorem for bounded mean curvature (if time permits)

(3) Application to General Relativity
  (a) Hawking mass and monotonicity under IMCF, compare G. Huisken and T. Ilmanen [4]
  (b) Applications to concepts of isoperimetric mass and Bartnik quasi-local mass
  (c) Partial results and speculations about the case of general initial data sets
  (d) Discussion of some open problems.

  Pre-lecture:
  (a) Asymptotically flat 3-manifolds and their role in General Relativity
  (b) Classical definition of ADM-mass on asymptotically flat 3-manifolds
  (c) Initial data sets, the dominant energy condition and non-negative scalar curvature on maximal initial data sets

References

[1] P. Daskalopoulos and G. Huisken, *Inverse mean curvature evolution of entire graphs*; Calc. Var. Partial Differ. Equ. 61 (2022), Paper No. 53

[2] C. Gerhardt, *Flow of nonconvex hypersurfaces into spheres*, J. Diff. Geom. 32 (1990), 299–314

[3] M. Heidusch, *Zur Regularitaet des inversen mittleren Kruemmungsflusses*, PhD thesis, Eberhard Karls Universitaet Tuebingen (2001)

[4] G. Huisken and T. Ilmanen, *The inverse mean curvature flow and the Riemannian Penrose inequality*, J. Diff. Geom. 59 (2001), 353-437

[5] G. Huisken and T. Ilmanen, *Higher regularity of the inverse mean curvature flow*, J. Diff. Geom. 80 (2008), 433–451

[6] G. Huisken and A. Polden, *Geometric evolution equations for hypersurfaces*, Calculus of variations and geoemtric evolution problems, Cetraro, Springer Lecture Notes 1713 (1999), 45-84

[7] K. Moore, *On the evolution of hypersurfaces by their inverse null mean curvature*, J. Diff. Geom. 98 (2014), 425–466